fact that the derivative of density with respect to velocity, which occurs in the function g of Eq. (1), is not correctly represented. They accept this incorrect representation on the grounds that this derivative does not occur explicitly in the basic physical equations of the problem, and probably mean, thereby, the integral version of these equations. They conclude from this, that a wildly oscillating representation of $d\rho/dV_c$ may be used, as long as its integral, ρ, approximates the correct $\rho(V_c)$. This could probably be justified by consideration of the integral form of the equations of motion, and we will accept it, although mathematicians might object to violent tinkering with coefficients of partial differential equations.

However, the application that Cahn and Garcia make of this principle is completely erroneous. In fact, they push things too far when they consider a piecewise continuous representation of density in function of velocity as shown on their Fig. 3, which leads to infinite values for the density derivative at the points of discontinuity. This is an essential fact which the authors appear to have missed. If one considers the piecewise continuous representation of Fig. 3 as the limit of a smooth curve with very large slope in small regions around the points of discontinuity, one is led to conclude from the second Eq. (1), that $\partial\phi_c/\partial V_c$ will be of a similar large order of magnitude. In the limiting case of a discontinuous density, one should therefore expect finite jumps in the potential. These jumps are not considered by the authors and their justification for the use of the transformation (7) is therefore not valid, as expected from our earlier arguments.

In the last paragraph of their section "Outline of Theory," Cahn and Garcia present another justification for their assumption about the validity of replacing the actual density by an approximation with wildly oscillating derivative. Although we do not reject this assumption and have shown that the error lies elsewhere, we cannot accept this justification, based on the consideration of the electric analogy tank with variable depth. The equation solved by the analogy is

$$(\partial/\partial x)(h\,\partial V/\partial x) + (\partial/\partial y)(h\,\partial V/\partial y) = 0 \qquad (1)$$

where V is the electric potential and h the depth, a function of the horizontal coordinates x,y. This function must, however, be slowly varying for (1) to be valid. If one considers, with Cahn and Garcia, a sawtooth shaped bottom, for which the depth would contain a wildly oscillating contribution, then a thin layer of three dimensional potential field will develop near the bottom. Outside this layer, a nearly two dimensional potential field will exist, satisfying the equation

$$(\partial/\partial x)(\bar{h}\,\partial V/\partial x) + (\partial/\partial y)(\bar{h}\,\partial V/\partial y) = 0 \qquad (2)$$

where $\bar{h}$ is a smoothed depth distribution involving only slowly varying components. These results can be derived by analyzing the solution of the three-dimensional Laplace equation

$$\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2 = 0 \qquad (3)$$

with boundary conditions representing the wavy bottom.

Cahn and Garcia appear to believe that the analogy will yield a solution of (1) with wildly oscillating h, but this is incorrect. Their last argument therefore is not relevant to the discussion of their basic assumption.

References

[1] Cahn, M. S. and Garcia, J. R., "Transonic Airfoil Design," *Journal of Aircraft*, Vol. 8 No. 2, Feb. 1971, pp. 84–88.

Reply by Authors to J. Smolderen

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OUR program has proven very useful for the design of transonic airfoils. Using our method airfoils can be designed to give desired characteristics at transonic speeds, and experiments have shown the resulting design to be sufficiently useful for engineering application. In addition, the design process using our method is extremely rapid; the program runs in only a few seconds on IBM 360.

It would be academically interesting to know exactly what approximations, if any, are involved in our computing process. We therefore would like to thank J. Smolderen for his comments. We feel his observations will be very helpful to our eventual complete understanding of the transonic mixed flow problem.

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